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Target Tracking with Multiple Transmitters in Clutter
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Joint Waveform and Cartesian Estimate Selection for Multistatic Target Tracking with Multiple Transmitters in Clutter

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Abstract

Converting multistatic radar measurements into Cartesian estimates of target position and velocity allows a linear Kalman filter, but the accuracy of each estimate depends on the transmitted waveform and on the geometry. Joint waveform and Cartesian estimate selection has so far been studied for one transmitter and without clutter. We extend it to several transmitters and to tracking in clutter. A new Cartesian estimate is formed from the bistatic ranges of two transmitters at one receiver, with a closed-form solution and an unscented-transform covariance. In clutter, candidate estimates formed from pairs of validated detections are combined by a probabilistic data association (PDA) filter, and the selection cost includes a clutter degradation factor computed by Monte Carlo integration. Simulations with Gaussian linear frequency-modulated (LFM) pulses show that a second transmitter reduces the position root mean square error (RMSE) by about 20% without clutter, mostly through its own channels; the new estimate adds about 1% in position and 5% in velocity. With the first-order covariance, 10.5% of the tracks are lost. In clutter, the reduction over the tracks kept is about 22% and 30% at the lowest and highest densities (200 runs) and 21% at the middle density (1000 runs), but when lost tracks are counted through an error cutoff it is significant only at the lowest density. The clutter factor reduces the position error by up to

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30%. An unscented PDA filter on the raw measurements of the same two channels, chosen by the proposed selection, is 11% to 34% more accurate, which is the price of keeping a linear filter. A third transmitter adds little, and error maps show that the placement of the nodes matters strongly.

Keywords: multistatic radar; adaptive waveform selection; Cartesian estimate; probabilistic data association; unscented transform; target tracking.

الاختيار المشترك للموجة المرسله والتقدير الديكارتي لتتبع الأهداف في الرادار متعدد المواقع بعدة مرسلات في وجود الصدى غير المرغوب

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الملخص

يتيح تحويل قياسات الرادار متعدد المواقع إلى تقديرات ديكارتية لموقع الهدف وسرعته استخدام مرشح كالمان الخطي، غير أن دقة كل تقدير تعتمد على الموجة المرسله وعلى هندسة المنظومة. وقد دُرِس الاختيار المشترك للموجة والتقدير الديكارتي لمرسل واحد وفي غياب الصدى غير المرغوب (Clutter)، وتوسَّع هذه الورقة ليشمل عدة مرسلات والتتبع في وجود الصدى. تقدم الورقة تقديراً ديكارتيّاً جديداً من المدى ثنائي الموقع لمرسلين عند مستقبل واحد، بحل مغلق الصيغة ومصفوفة تغاير محسوبة بالتحويل غير المعطّر. وفي وجود الصدى تُكوّن تقديرات مرشحة من أزواج الكشف المقبولة وتُدمج بمرشح ربط البيانات الاحتمالي، وتتضمن دالة الاختيار معامل تدهور يُحسب بتكامل مونت كارلو. وتُظهر المحاكاة بنبضات غاوسية معدلة خطياً في التردد أن المرسل الثاني يخفض الجذر التربيعي لمتوسط مربع خطأ الموقع بنحو 20% في غياب الصدى، ومعظم ذلك من قنواته، إذ لا يضيف التقدير الجديد إلا نحو 1% في الموقع و5% في السرعة، بينما يُفقد 10.5% من المسارات مع تقريب الرتبة الأولى. وفي وجود الصدى يبلغ الانخفاض في

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المسارات غير المفقودة نحو 22% و30% عند الكثافتين الدنيا والعليا (200 تشغيلية) و21% عند الكثافة الوسطى (1000 تشغيلية)، لكنه لا يبقى معنوياً إلا عند الكثافة الدنيا عند احتساب المسارات المفقودة بخطأ محدود بسقف. ويخفض معامل الصدى في دالة الاختيار خطأ الموقع حتى 30%. ويتفوق مرشح كالمان غير المعطر مع ربط البيانات الاحتمالي على القياسات الخام للقناتين نفسيهما اللتين يختارهما النهج المقترح بنسبة 11% إلى 34%، وهذا ثمن الإبقاء على مرشح خطي. ولا يضيف المرسل الثالث إلا القليل، وتبين خرائط الخطأ أن مواقع العقد تؤثر بقوة في النتائج. الكلمات المفتاحية: الرادار متعدد المواقع؛ الاختيار التكيفي للموجة؛ التقدير الديكارتي؛ ربط البيانات الاحتمالي؛ التحويل غير المعطر؛ تتبع الأهداف.

1. Introduction

A multistatic radar has transmitters and receivers at different sites. Each transmitter–receiver pair forms a bistatic channel that measures the bistatic range, the bistatic range rate and, with an antenna array, the angle of arrival of the echo. The spatial diversity of the channels improves coverage and accuracy, but the measurements are nonlinear functions of the target state (Dhulashia & Ritchie, 2024; Farina, 1986; Haimovich et al., 2008; Willis & Griffiths, 2007). One way to handle this nonlinearity is to convert measurements from two channels into an estimate of the target position and velocity in Cartesian coordinates and pass this estimate to a linear Kalman filter with an identity measurement matrix (Lerro & Bar-Shalom, 1993; Nguyen et al., 2015a; Wang & Bao, 2021). This conversion moves the nonlinearity out of the filter, but the accuracy of the converted estimate then depends on the measurements used, the geometry, and the transmitted waveform. The waveform affects tracking because the range and range-rate accuracy depend on it. Kershaw and Evans (1994) showed that the waveform can be chosen at each scan to minimize the predicted tracking error and later included clutter through the probabilistic data association (PDA) filter (Kershaw & Evans, 1996, 1997). More recent studies have chosen the waveform by online learning (Thornton et al., 2022) or together with the transmit resources of multistatic and networked radars (Shi et al., 2022; Wang et al.,

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2021). For multistatic radar, Nguyen et al. (2015b) selected the waveform adaptively and proposed the joint selection of the waveform and the Cartesian estimate for one transmitter (Nguyen & Doğançay, 2019; Nguyen et al., 2015a). The authors' earlier conference paper followed this framework for a two-dimensional system with one transmitter, three receivers and Gaussian linear frequency-modulated (LFM) pulses, and reported the Cramér–Rao lower bounds (CRLB) of three Cartesian estimates (Khalifa & Abousetta, 2025). The present paper extends that conference paper, which considered one transmitter without clutter; Sections 3.2, 4.2, 4.3 and 6, and the UT covariance in Section 3.3, are new.

These works leave three questions open. First, practical multistatic systems often have several transmitters (Godrich et al., 2010; Haimovich et al., 2008; Shi et al., 2022), but the joint selection has been formulated for one. Nguyen and Doğançay (2019) studied several transmitters for localization geometry, and later work examined sensor placement and receiver selection (Liang et al., 2021; Zhu et al., 2021), but not estimate selection in tracking. Second, selection with Cartesian estimates assumes that every measurement comes from the target, while clutter changes waveform comparisons (Feng et al., 2024; Huang et al., 2024; Kershaw & Evans, 1996, 1997). Third, the effect of radar-node positions on selection has not been quantified.

Each scan in this paper uses one Cartesian estimate, and therefore two bistatic channels, even when three transmitters and three receivers give nine channels. This keeps the load on the fusion centre independent of the number of nodes (Khalifa & Abousetta, 2025) and keeps the update linear. The price is that most channels are unused in a given scan, so an extra transmitter only adds channel pairs to choose from; this is why a third transmitter adds little (Section 6.1). Since a nonlinear filter on the raw measurements needs no conversion, Section 6 also compares the scheme with an unscented PDA filter on the same two channels.

This paper addresses these questions. Its contributions are as follows:

1. A Cartesian estimate that combines the bistatic ranges of two transmitters at one receiver is introduced, with a closed-form

- solution and a rule for resolving its ambiguity. We compute its covariance with the unscented transform (UT) and show that the first-order covariance is inadequate.
2. The joint selection is extended to several transmitters and to tracking in clutter. A nonparametric PDA filter combines candidate estimates formed from pairs of validated detections, and the selection cost includes a clutter degradation factor computed by Monte Carlo integration.
 3. The scheme is evaluated by Monte Carlo simulation with paired statistical comparisons, against a nonlinear baseline and with an error measure that counts lost tracks, and the effect of the positions of a transmitter and of a receiver is mapped.

2. System Model

2.1 Geometry and Target Motion

The system has N_t transmitters at known positions $\mathbf{t}_i$ and $N_r = 3$ receivers at known positions $\mathbf{r}_j$ in two dimensions. Transmitter 1 and the three receivers are placed as in Khalifa and Abousetta (2025): $\mathbf{t}_1 = [0,0]^T \text{km}$, $\mathbf{r}_1 = [20,0]^T \text{km}$, $\mathbf{r}_2 = [10,20]^T \text{km}$ and $\mathbf{r}_3 = [0,15]^T \text{km}$. Additional transmitters are placed at $\mathbf{t}_2 = [20,20]^T \text{km}$ and $\mathbf{t}_3 = [25,10]^T \text{km}$ (Fig. 1).

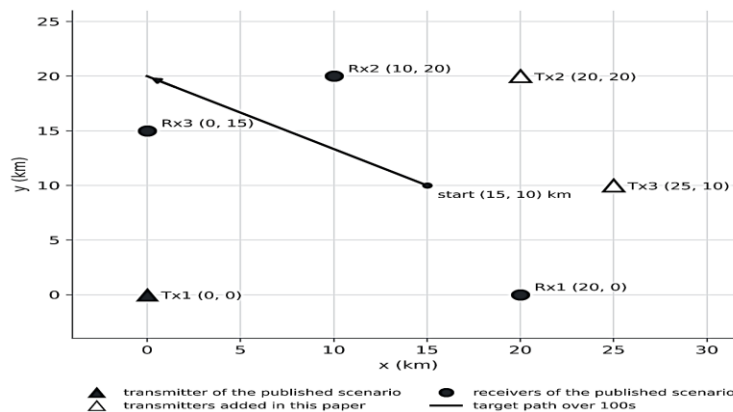


Fig. 1. Radar configuration and target path. Filled symbols: configuration of Khalifa and Abousetta (2025); open triangles: transmitters added in this paper.

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The transmitters use separable waveforms, and the nodes are synchronized. The target state is $\mathbf{x}_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T$. It follows the nearly constant-velocity model

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{w}_k \quad (1a)$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{I}_2 & T\mathbf{I}_2 \\ \mathbf{0} & \mathbf{I}_2 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} (T^2/2)\mathbf{I}_2 \\ T\mathbf{I}_2 \end{bmatrix} \quad (1b)$$

where T is the scan time and $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \sigma_a^2 \mathbf{I}_2)$, so that $\mathbf{Q} = \sigma_a^2 \mathbf{G}\mathbf{G}^T$ (Bar-Shalom et al., 2001).

2.2 Measurements and Waveform

Channel (i, j) provides the bistatic range, the bistatic range rate and the angle of arrival at the receiver,

$$R_{ij} = \|\mathbf{p} - \mathbf{t}_i\| + \|\mathbf{p} - \mathbf{r}_j\| \quad (2a)$$

$$\dot{R}_{ij} = (\mathbf{u}_{T,i} + \mathbf{u}_{R,j})^T \mathbf{v} \quad (2b)$$

$$\theta_j = \text{atan2}(y - y_{r,j}, x - x_{r,j}) \quad (2c)$$

where $\mathbf{p}$ and $\mathbf{v}$ are the target position and velocity, and $\mathbf{u}_{T,i}$ and $\mathbf{u}_{R,j}$ are unit vectors pointing to the target from the transmitter and from the receiver. Each transmitter sends a Gaussian-envelope LFM pulse defined by a pulse-length parameter λ and a frequency sweep F . For the chirp rate $\gamma = F/(2T_s)$ and the effective duration $T_s = 7.4338\lambda$, the CRLB on the delay and Doppler errors at a signal-to-noise ratio (SNR) η is (Nguyen & Doğançay, 2019; Nguyen et al., 2015b)

$$\mathbf{N} = \frac{1}{\eta} \begin{bmatrix} 2\lambda^2 & -4\gamma\lambda^2 \\ -4\gamma\lambda^2 & \frac{1}{2\pi^2\lambda^2} + 8\gamma^2\lambda^2 \end{bmatrix} \quad (3)$$

and the covariance of the range and range rate $(R_{ij}, \dot{R}_{ij})$ is obtained by multiplying the delay and Doppler terms by the speed of light c and by c/f_c , respectively, where f_c is the carrier frequency. Channel (i, j) has the SNR $\eta_{ij} = (R_0^2 / (R_{T,i} R_{R,j}))^2$ (Khalifa & Abousetta,

2025; Nguyen & Doğançay, 2019). Long pulses favour the range rate at the expense of the range, and the off-diagonal term of (3) makes the two errors strongly correlated. The angle error is zero-mean Gaussian with variance $\sigma_\theta^2/\eta_{ij}$ and does not depend on the waveform (Nguyen & Doğançay, 2019).

2.3 Detection and Clutter

The target is detected on each channel with probability P_D . False detections follow a Poisson model and are uniform over the space of the three measured values of each channel, with density ρ per $\text{m} \cdot (\text{m/s}) \cdot \text{rad}$. This extends the model of Nguyen and Doğançay (2019), which is uniform over range and range rate, to include the angle. Detections of different channels and scans are independent.

3. Cartesian Estimates

3.1 Estimates with One Transmitter

A Cartesian estimate is built from four measurements of two channels: two of them give the position, and the two range rates give the velocity. With transmitter i and receivers j and l , three such estimates are possible (Table 1; Khalifa & Abousetta, 2025; Nguyen & Doğançay, 2019; Nguyen et al., 2015a). For $\mathbf{y}_1$, the position is the point where the ellipse of R_{ij} meets the line of sight θ_j .

$$\text{Using } \mathbf{u} = [\cos\theta_j, \sin\theta_j]^T \text{ and } \mathbf{d} = \mathbf{r}_j - \mathbf{t}_i,$$

$$\hat{\mathbf{p}} = \mathbf{r}_j + \frac{R_{ij}^2 - \|\mathbf{d}\|^2}{2(R_{ij} + \mathbf{d}^T \mathbf{u})} \mathbf{u} \quad (4)$$

The estimate $\mathbf{y}_2$ uses receiver l in the same way, and $\mathbf{y}_3$ is the crossing point of the two lines of sight. For all three, the velocity is found from the two range rates,

$$\hat{\mathbf{v}} = \mathbf{B}^{-1} \begin{bmatrix} \dot{R}_{ij} \\ \dot{R}_{il} \end{bmatrix} \quad (5a)$$

$$\mathbf{B} = \begin{bmatrix} (\mathbf{u}_{T,i} + \mathbf{u}_{R,j})^T \\ (\mathbf{u}_{T,i} + \mathbf{u}_{R,l})^T \end{bmatrix} \quad (5b)$$

evaluated at $\hat{\mathbf{p}}$. Three receivers form three receiver pairs, so each transmitter offers nine estimates, as in the tracking example of Nguyen and Doğançay (2019); Khalifa and Abousetta (2025) reported the bounds for a single pair.

3.2 Estimate with Two Transmitters

With two transmitters i and i' , a fourth estimate $\mathbf{y}_4$ uses the ranges R_{ij} and $R_{i'j}$ at the same receiver j , together with the two range rates $\dot{R}_{ij}$ and $\dot{R}_{i'j}$. The two ellipses share the focus $\mathbf{r}_j$ (Fig. 2(d)).

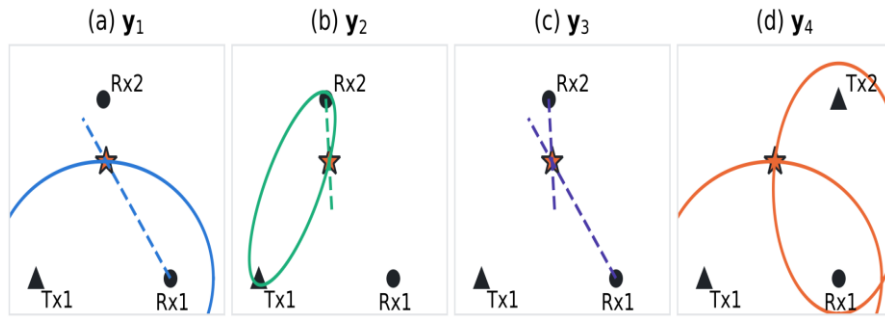


Fig. 2. The four types of Cartesian estimate: (a) range and angle at Rx1; (b) range and angle at Rx2; (c) angles at Rx1 and Rx2; (d) ranges of two transmitters at Rx1. Solid lines: constant-range ellipses; dashed lines: measured directions; star: target.

Let $\mathbf{q} = \mathbf{p} - \mathbf{r}_j$, $d_j = \|\mathbf{q}\|$ and $\mathbf{a}_i = \mathbf{t}_i - \mathbf{r}_j$. Since the transmitter–target distance is $R_{ij} - d_j$, squaring $\|\mathbf{q} - \mathbf{a}_i\| = R_{ij} - d_j$ gives

$$\mathbf{a}_i^T \mathbf{q} = 1/2 (\|\mathbf{a}_i\|^2 - R_{ij}^2) + R_{ij} d_j \quad (6)$$

which is linear in $\mathbf{q}$. The equations of the two transmitters give

$\mathbf{q} = \mathbf{c} + \mathbf{f} d_j$, with $\mathbf{c}$ and $\mathbf{f}$ obtained by inverting the 2×2 matrix $\mathbf{A} = [\mathbf{a}_i, \mathbf{a}_{i'}]^T$. Substituting into $\|\mathbf{q}\|^2 = d_j^2$ gives

$$(\mathbf{f}^T \mathbf{f} - 1) d_j^2 + 2 \mathbf{c}^T \mathbf{f} d_j + \mathbf{c}^T \mathbf{c} = 0 \quad (7)$$

A root is admissible if $0 < d_j < \min(R_{ij}, R_{i'j})$. If both roots are admissible, the one closer to the predicted position is kept.

Combinations in which the two transmitters and the receiver are collinear make $\mathbf{A}$ singular and are excluded. The velocity follows from (5) with the unit vectors of transmitters i and i' . Unlike $\mathbf{y}_1$ to $\mathbf{y}_3$, the estimate $\mathbf{y}_4$ needs no angle measurement.

Table 1. Cartesian estimates for transmitter i and receiver pair (j, l) , or transmitters i, i' and receiver j

Estimate	Position from	Velocity from	Waveform-dependent
$\mathbf{y}_1$	R_{ij}, θ_j	$\dot{R}_{ij}, \dot{R}_{il}$	position, velocity
$\mathbf{y}_2$	R_{il}, θ_l	$\dot{R}_{ij}, \dot{R}_{il}$	position, velocity
$\mathbf{y}_3$	θ_j, θ_l	$\dot{R}_{ij}, \dot{R}_{il}$	velocity
$\mathbf{y}_4$ (new)	$R_{ij}, R_{i'j}$	$\dot{R}_{ij}, \dot{R}_{i'j}$	position, velocity

3.3 Covariance of the Estimates

Let $\mathbf{C}_z$ be the covariance of the four measurements and $\mathbf{H}_z$ the matrix of their gradients with respect to the state. The usual first-order covariance of an estimate is (Khalifa & Abousetta, 2025; Nguyen et al., 2015a)

$$\mathbf{C}_y \approx \mathbf{H}_z^{-1} \mathbf{C}_z \mathbf{H}_z^{-T} \quad (8)$$

With single-scan range errors of the order of a kilometre and strongly correlated range and range-rate errors, (8) can underestimate the error of the conversion (Bordonaro et al., 2019; Lerro & Bar-Shalom, 1993; Wang & Bao, 2021). The UT passes $2n + 1$ sigma points through the exact conversion $\mathbf{g}$ (Asif et al., 2023; Julier & Uhlmann, 2004),

$$\mathbf{C}_y^{\text{UT}} = \sum_{s=-n}^n w_s \boldsymbol{\delta}_s \boldsymbol{\delta}_s^T, \quad \boldsymbol{\delta}_s = \mathbf{g}(\mathbf{Z}_s) - \hat{\mathbf{x}} \quad (9a)$$

$$\mathbf{Z}_{\pm s} = \mathbf{z}_0 \pm \left[\sqrt{(n + \kappa) \mathbf{C}_z} \right]_s \quad (9b)$$

with $n = 4$, $\kappa = 1$, $\mathbf{Z}_0 = \mathbf{z}_0 = \mathbf{h}(\hat{\mathbf{x}})$, $w_0 = \kappa/(n + \kappa)$ and $w_{\pm s} = 1/(2(n + \kappa))$, where $[\cdot]_s$ is the s th column of a matrix square root. The spread is taken about $\hat{\mathbf{x}} = \mathbf{g}(\mathbf{z}_0)$, so that a conversion bias is included.

The two covariances were compared for $\mathbf{y}_4$. At five positions along the target path, for each receiver where $\mathbf{y}_4$ was admissible and for 40 random pairs of waveforms, 3000 noisy measurement vectors were converted and the normalized errors $\epsilon^T \mathbf{C}_y^{-1} \epsilon$ were computed. For a correct covariance, their median is 3.36. Over the 480 cases, the median was 4.3 with (8) and 2.7 with (9), and the median exceeded twice the correct value in 16% of the cases with (8) and in 1% with (9). The first-order covariance therefore tends to underestimate the error of $\mathbf{y}_4$, while the UT is slightly conservative. An estimate is offered to the selection only if, at the predicted state, the two position curves cross at an angle of at least 30° , each range-rate direction in (5) has a length of at least 0.7 (a bistatic angle below about 140°), and the two range-rate directions cross at an angle of at least 30° . These thresholds were set by trial. With 200 runs of S8 at $\rho = 3 \times 10^{-5}$, angle limits of 20° and 40° changed the position RMSE over the runs kept by both settings by -0.1% (-0.4% to 0.2%) and 1.6% (-0.4% to 5.1%), and length limits of 0.6 and 0.8 by 0.0% (-0.1% to 0.2%) and -0.7% (-3.0% to 0.7%). Losses were 3.0%, 4.0%, 2.5% and 2.5%, against 2.5%; the extra losses with the 40° limit raise the cutoff error of Section 5 by 13.4% (0.5% to 40.4%). Moderate changes of the thresholds therefore have little effect, although a stricter angle limit can cost some tracks.

4. Selection and Tracking

4.1 Selection without Clutter

With a Cartesian estimate as the measurement, the Kalman update has $\mathbf{H} = \mathbf{I}$, the innovation covariance $\mathbf{S} = \mathbf{P}_{k+1|k} + \mathbf{C}_y$ and the gain $\mathbf{K} = \mathbf{P}_{k+1|k} \mathbf{S}^{-1}$. The estimate m and the waveforms $\{\psi_i\}$ of the transmitters it uses are selected as (Khalifa & Abousetta, 2025; Nguyen et al., 2015a)

$$J(m, \{\psi_i\}) = \text{tr} \left(\mathbf{P}_{k+1|k}^{-1} + \mathbf{C}_y^{-1}(m, \{\psi_i\}) \right)^{-1} \quad (10a)$$

$$(m^*, \{\psi_i^*\}) = \arg \min_{m \in \mathcal{M}_{k+1}, \psi_i \in \mathcal{W}} J(m, \{\psi_i\}) \quad (10b)$$

where $\mathcal{M}_{k+1}$ is the set of admissible estimates, $\mathcal{W}$ the waveform library, and i ranges over the transmitters used by estimate m . For $\mathbf{y}_1$ to $\mathbf{y}_3$ the search covers the waveforms of one transmitter; for $\mathbf{y}_4$ it covers all pairs of waveforms of two transmitters. Each estimate uses exactly two channels, so only these channels report to the fusion centre at each scan, whatever the number of transmitters.

4.2 Selection Cost in Clutter

With clutter, the update is a PDA update, whose expected covariance is approximated by the modified Riccati equation (Bar-Shalom et al., 2009; Kershaw & Evans, 1996; Nguyen & Doğançay, 2019),

$$\bar{\mathbf{P}} = \mathbf{P}_{k+1|k} - q \mathbf{P}_{k+1|k} (\mathbf{P}_{k+1|k} + \mathbf{C}_y)^{-1} \mathbf{P}_{k+1|k} \quad (11)$$

where the degradation factor $q \in (0,1]$ depends on P_D and on the mean number $\mu = \rho V$ of false detections in a gate of volume V . Since an estimate is correct only if the detections of both its channels are correct, its factor is approximated by the product of the channel factors,

$$q(m, \{\Psi_i\}) = q_c(P_D, \rho V_{c_1}) q_c(P_D, \rho V_{c_2}) \quad (12a)$$

$$V_c = \frac{4\pi}{3} g^3 |\mathbf{S}_c|^{\frac{1}{2}} \quad (12b)$$

where $\mathbf{S}_c$ is the innovation covariance of the three measurements of channel c and g is the gate threshold in standard deviations. The waveform therefore affects the cost both through the accuracy of the estimate and through the size of the gates (Kershaw & Evans, 1997). The selection minimizes $\text{tr } \bar{\mathbf{P}}$ instead of J in (10).

The approximation of Kershaw and Evans (1996) for q applies to two-dimensional measurements. For three measurements, q_c was computed by Monte Carlo integration, as noted by Nguyen and Doğançay (2019). In the whitened innovation space, the target measurement is drawn inside the gate with probability $P_D P_G$ and Poisson false detections are spread uniformly in the gate; the PDA

weights β_l and the combined innovation $\bar{\mathbf{v}}$ are computed for each draw (Bar-Shalom et al., 2009), and

$$q = E \left[1 - \beta_0 - \frac{1}{n} \left(\sum_l \beta_l \|\mathbf{v}_l\|^2 - \|\bar{\mathbf{v}}\|^2 \right) \right] \quad (13)$$

The expectation was evaluated with 60000 draws at 26 values of μ between 0 and 20 and interpolated. For two measurements and $P_D = 0.9$, the same computation agrees with the approximation of Kershaw and Evans (1996) within about 0.015.

4.3 Tracking in Clutter

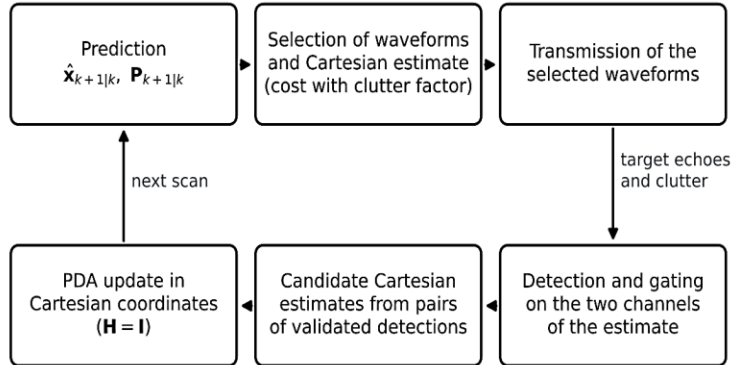
The detections of the two channels of the selected estimate are gated with a gate of four standard deviations. Each pair of validated detections, one from each channel, gives a candidate Cartesian estimate. The candidates are gated in Cartesian coordinates with the 99.9% threshold $g_y^2 = 18.47$ of the chi-square distribution with four degrees of freedom. The M candidates that remain are combined by a nonparametric PDA filter with $\mathbf{H} = \mathbf{I}$, in which the clutter density is replaced by M/V_y (Bar-Shalom et al., 2009; Nguyen & Doğançay, 2019):

$$\beta_l = \frac{e_l}{b + \sum_{n=1}^M e_n}, \quad e_l = \exp(-1/2 \mathbf{v}_l^T \mathbf{S}^{-1} \mathbf{v}_l) \quad (14a)$$

$$b = \frac{M}{V_y} |2\pi \mathbf{S}|^{\frac{1}{2}} \frac{1 - P_{D,y} P_{G,y}}{P_{D,y}} \quad (14b)$$

where $V_y = (\pi^2/2) g_y^4 |\mathbf{S}|^{1/2}$, $P_{G,y} = 0.999$ and $P_{D,y} = (P_D P_{G,c})^2$ is the probability that both target detections are present and validated. The state and covariance are updated with the standard PDA equations (Bar-Shalom et al., 2009). If no candidate remains, the track is predicted. Fig. 3 summarizes one scan.

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Only the measurements of the two channels used by the selected estimate are processed at each scan.

Fig. 3. Processing at each scan.

5. Simulation Setup

Table 2 lists the parameters. The radar, waveform, target and angle parameters follow Khalifa and Abousetta (2025). The library is a grid of 25 waveforms covering the ranges of Khalifa and Abousetta (2025). We ran each case 200 times, with 100 scans per run.

Table 2. Simulation parameters

Parameter	Value
Carrier frequency f_c	12.5GHz
Pulse-length parameter λ	40, 50, 60, 70, 80 μ s
Frequency sweep F	0.1, 0.2, 0.3, 0.4, 0.5MHz
Fixed waveform	$\lambda = 60\mu$ s, $F = 0.3$ MHz
Reference range R_0 ; angle constant σ_θ	70km; 0.2rad
Initial position and velocity	$[15, 10]^T$ km; $[-150, 100]^T$ m/s
Scan time T ; scans; process noise σ_a	1s; 100; 2m/s ²
Initial covariance $\mathbf{P}_0$	diag($10^5, 10^5, 10^2, 10^2$) (Nguyen & Doğançay, 2019)
Detection probability P_d	0.9 per channel
Clutter density ρ	0, 10^{-5} , 3×10^{-5} , 10^{-4} per m $\cdot$ (m/s) $\cdot$ rad
Gates	4 standard deviations per channel; $g_y^2 = 18.47$

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In each run, all strategies use the same target path and initial estimate, and their noise and clutter come from random sequences with the same seeds. Table 3 lists the strategies compared. In the fixed strategies, the estimate and the waveform do not change; in the random strategy, the waveforms are drawn at random at each scan and the estimate is then selected.

Table 3. Strategies compared

Strategy	Tx	Waveform	Estimate	Covariance	Clutter in cost
S1: one Tx, fixed	1	fixed	y_1 (Rx1, Rx2)	(8)	—
S2: one Tx, adaptive (Khalifa & Abousetta, 2025)	1	selected	selected	(8)	yes
S3: one Tx, adaptive, UT	1	selected	selected	(9)	yes
S3': one Tx, adaptive, UT, cost without clutter	1	selected	selected	(9)	no
S4: two Tx, fixed	2	fixed	y_4 (Rx1)	(8)	—
S5: two Tx, random waveforms	2	random	selected	(9)	yes
S6: two Tx, adaptive, first order	2	selected	selected	(8)	yes
S7: two Tx, cost without clutter	2	selected	selected	(9)	no
S8: two Tx, proposed	2	selected	selected	(9)	yes
S9: three Tx, proposed	3	selected	selected	(9)	yes
S10: two Tx, proposed without y_4	2	selected	selected (y_1 – y_3)	(9)	yes
S11: two Tx, UKF-PDA on raw measurements	2	as S8	none	UKF	yes

Strategy S2 extends the scheme of Khalifa and Abousetta (2025) to all receiver pairs; without clutter, its factor q is the same for all candidates and does not change the selection. A track is lost if the target candidate falls outside the gates in more than four consecutive scans in which the target was detected on both channels, following Nguyen and Doğançay (2019). The root mean square error (RMSE) is computed over the tracks that were not lost, and the percentage of

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lost tracks is given next to it (Nguyen & Doğançay, 2019). Two strategies are compared by the change in pooled RMSE over the runs that neither lost, with a 95% bootstrap interval from 5000 resamples (Efron & Tibshirani, 1993), and their losses by a two-sided sign test.

Table 3 also lists two further strategies. S10 is S8 without y_4 . S11 uses the channels and waveforms chosen by S8 at each scan but updates the track with an unscented Kalman filter and a nonparametric PDA on the six raw measurements of the two channels (Bar-Shalom et al., 2009; Julier & Uhlmann, 2004); each pair of validated detections is one candidate, gated at the 99.9% chi-square level with six degrees of freedom. S11 therefore relies on the selection of S8, whose cost is computed for the Cartesian estimates, and is not a complete alternative scheme. Because an RMSE over the tracks kept favours strategies that lose more tracks, we also report a cutoff error: the position error at each scan is limited to $c = 500$ m, and the root mean square is taken over all runs, including the lost ones. For one target, this is the OSPA distance of order 2 with cutoff c (Schuhmacher et al., 2008).

6. Results and Discussion

6.1 Without Clutter

Table 4 gives the results without clutter.

Table 4. Results without clutter ($P_D = 0.9$, 200 runs). Position RMSE with an approximate 95% interval

Strategy	Position RMSE (m)	Velocity RMSE (m/s)	Lost (%)	Selection time (ms/scan)
S1	155.3 ± 6.4	10.12	0	—
S2	35.9 ± 2.0	5.05	0	3.5
S3	35.6 ± 2.0	5.17	0	4.2
S4	75.5 ± 3.0	5.65	0	—
S5	36.0 ± 2.0	4.67	0	8.0
S6	36.6 ± 1.9	4.98	10.5	10.5
S8	28.6 ± 2.0	4.38	0	20.8
S9	28.1 ± 2.0	4.27	0	36.5
S10	28.9 ± 2.0	4.59	0	—
S11	25.3 ± 2.1	3.25	0	—

Selection times of S10 and S11 are given in Section 6.1; they were measured in a separate session.

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The adaptive selection with one transmitter (S2) reduces the position RMSE of the fixed configuration from 155m to 35.9m. For one transmitter, the first-order and UT covariances give practically the same result (S2 and S3); their paired difference is -1.0% (interval -2.1% to -0.02%). S3' and S7 are not listed because, without clutter, their cost is the same as that of S3 and S8, and they give the same results. With a second transmitter and the UT covariance (S8), the position RMSE decreases to 28.6m and the velocity RMSE to 4.38m/s. Relative to S3, the paired change in position RMSE is -19.6% (95% interval: -23.6% to -16.1%). Figure 4 shows that S2 and S3 behave almost identically, while the two-transmitter strategy gives lower error until about 65s, when the target is still relatively far from receivers 2 and 3.

Two further strategies separate the sources of this gain. Without y_4 (S10), the position RMSE is 28.9 m, a change of -18.7% (-22.5% to -15.2%) from S3, and adding y_4 (S10 $\rightarrow$ S8) gives only -1.1% (-1.8% to -0.5%) more, although the velocity RMSE falls from 4.59 to 4.38 m/s. Most of the gain therefore comes from the single-transmitter estimates on the channels of the second transmitter. The nonlinear baseline S11 reaches 25.3 m and 3.25 m/s, a change of -11.4% (-17.4% to -4.9%) from S8. The Cartesian estimate keeps four of the six measurements; the lost information is the price of a linear update and of a selection cost in closed form. Measured in one session on the same computer (40 runs), the update of S11 took 0.97 ms per scan against 0.80 ms for S8 without clutter, and 1.05 ms against 0.85 ms at the highest density, so S11 costs about 0.2 ms more per scan. The selection, which S11 shares with S8, took 29.1 ms per scan in this session (20.8 ms in Table 4, measured in an earlier session) and 11.0 ms for S10, which has fewer candidates. The selection therefore dominates the run time of all three, and all are small next to the 1 s scan interval.

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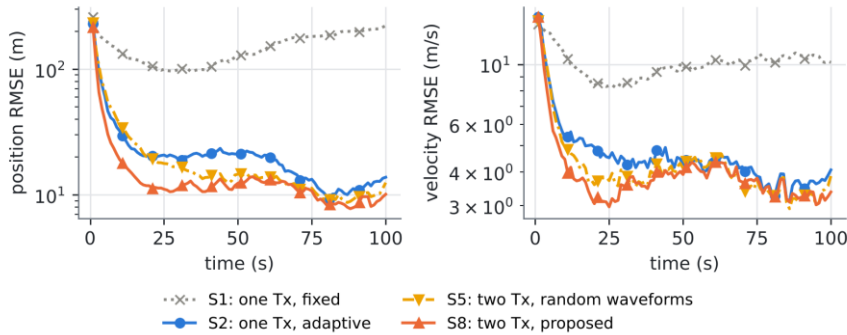


Fig. 4. Position and velocity RMSE over time without clutter.

Using the first-order covariance with the same selection scheme (S6) leads to a 10.5% track-loss rate and does not improve accuracy over the one-transmitter case. The correct candidate also falls outside the Cartesian gate more often, resulting in no update in 26.5% of the scans, compared with 19.1% for S8. This is consistent with the underestimated covariance of $\mathbf{y}_4$ discussed in Section 3.3: S6 selects $\mathbf{y}_4$ in about 70% of the scans, and S8 in only 32%.

The waveform choice matters as well. With random waveforms (S5), the position RMSE is 36.0 m, and moving from S5 to S8 lowers it by 20.6% (95% interval: -24.6% to -17.1%). The fixed two-transmitter configuration (S4) does much worse, with a position RMSE of 75.5 m. A third transmitter (S9) changes the RMSE by only -1.7% (-4.7% to 1.2%), which is not significant, while its selection time rises from 20.8 ms to 36.5 ms per scan. Both times are small next to the 1 s scan interval. The small gain is expected: each scan still uses only two channels, and a third transmitter only adds channel pairs to choose from (Section 1).

6.2 In Clutter

Table 5 and Figure 5 summarize the results in clutter. For S8, the mean number of false detections in a channel gate is 0.10, 0.31 and 1.27 at the three densities, against 0.59, 2.06 and 12.70 for S1, which loses 25.5% of its tracks at the highest density. Without the clutter factor, this number rises from 1.27 to 3.67 with two transmitters and from 2.79 to 6.66 with one.

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Table 5. Position RMSE (m) over the tracks kept, with the percentage of lost tracks in brackets (200 runs)

Strategy	$\rho = 10^{-5}$	$\rho = 3 \times 10^{-5}$	$\rho = 10^{-4}$
S1	172.2 (9.0)	212.7 (7.5)	291.5 (25.5)
S2	48.3 (3.5)	61.9 (4.5)	95.5 (8.0)
S3	48.1 (2.5)	62.0 (5.0)	104.7 (6.0)
S3'	56.9 (1.5)	75.5 (5.0)	131.3 (10.5)
S4	89.5 (2.5)	112.2 (10.5)	172.8 (36.5)
S5	45.6 (1.0)	58.3 (3.0)	85.9 (8.5)
S6	—	55.0 (10.0)	—
S7	41.1 (2.5)	61.8 (4.0)	107.7 (10.0)
S8	37.9 (1.0)	54.4 (2.5)	77.7 (6.0)
S9	38.8 (0.5)	47.1 (5.0)	75.7 (6.0)
S10	38.2 (1.0)	54.8 (2.0)	77.5 (5.5)
S11	33.2 (1.0)	35.5 (1.5)	54.2 (2.5)

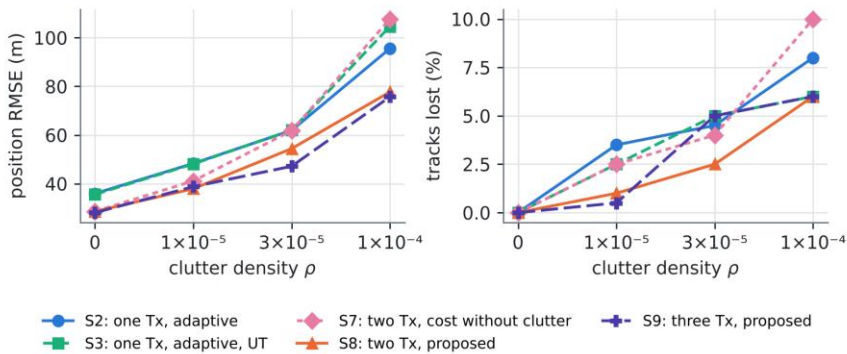


Fig. 5. Position RMSE and percentage of lost tracks against the clutter density.

Table 6 gives the paired comparisons. With 200 runs, the second transmitter reduces the position RMSE by about 22% at the lowest density and 30% at the highest; at the middle density the interval includes zero. The clutter factor is clearly useful at the highest density, where it reduces the RMSE by about 30% with two transmitters and 21% with one; at the lowest density the q values of the candidates are similar and its effect is smaller. Selected waveforms are 15–17% better than random ones at the lowest and highest densities. At the middle density, S9 seems better than S8 in Table 5 only because different lost runs are excluded (10 for S9 and 5 for S8); over the runs kept by both, S9 is 1.9% worse, which is not

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significant. S6, run only at the middle density, loses 10.0% of its tracks, against 5.0% for S3.

In clutter, removing y_4 (S10) changes the position RMSE by less than 1%, and all three intervals reach zero. S11 is more accurate than S8 at every density, by 13.8%, 30.2% and 33.5%, and loses as many or fewer tracks (1.0%, 1.5% and 2.5%, against 1.0%, 2.5% and 6.0%; not significant). The gap grows with the density, possibly because the six-dimensional gate uses both angles and rejects clutter pairs more easily.

Table 6. Paired change in position RMSE (%) with 95% interval (200 runs)

Comparison	$\rho = 10^{-5}$	$\rho = 3 \times 10^{-5}$	$\rho = 10^{-4}$
S3 $\rightarrow$ S8 (second Tx)	-22.2 (-29.6, -14.1)	-14.2 (-30.8, 5.8)	-29.6 (-40.7, -15.5)
S3' $\rightarrow$ S3 (clutter factor, one Tx)	-14.3 (-24.6, -1.4)	-10.9 (-21.8, 2.0)	-21.0 (-36.1, -2.4)
S7 $\rightarrow$ S8 (clutter factor, two Tx)	-7.0 (-19.9, 9.0)	-13.9 (-32.7, 9.9)	-29.6 (-41.6, -13.7)
S5 $\rightarrow$ S8 (selected waveforms)	-16.7 (-26.6, -5.5)	-7.1 (-25.0, 12.1)	-14.5 (-25.4, -2.3)
S8 $\rightarrow$ S9 (third Tx)	-0.4 (-3.1, 2.7)	1.9 (-3.6, 9.6)	-2.4 (-4.4, -0.7)
S10 $\rightarrow$ S8 (estimate y_4)	-0.7 (-1.8, 0.4)	-0.9 (-2.5, 0.0)	0.1 (-0.2, 0.5)
S8 $\rightarrow$ S11 (UKF-PDA)	-13.8 (-18.1, -9.6)	-30.2 (-40.2, -18.6)	-33.5 (-39.8, -26.7)

To resolve the middle density and the track losses, four strategies were repeated with 1000 runs at $\rho = 3 \times 10^{-5}$ (Table 7). The second transmitter reduces the position RMSE by 20.8% (-26.8% to -14.3%); its losses, 2.4% against 3.3%, are not significantly different ($p = 0.24$). The clutter factor reduces the RMSE by 16.6% with two transmitters and by 17.4% with one, and with one transmitter it also reduces the losses from 5.4% to 3.3% ($p = 0.013$); with two transmitters the reduction from 3.9% to 2.4% is at the edge of significance ($p = 0.063$).

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Table 7. Results with 1000 runs at $\rho = 3 \times 10^{-5}$

Strategy	Position RMSE (m)	Velocity RMSE (m/s)	Lost (%)
One Tx, UT, cost without clutter (S3')	79.8	6.11	5.4
One Tx, UT, cost with clutter (S3)	67.1	5.80	3.3
Two Tx, cost without clutter (S7)	62.7	5.03	3.9
Two Tx, proposed (S8)	53.3	4.77	2.4

Table 8 gives the cutoff error, which includes all runs. S11 has the lowest error at every density, and the gains of the proposed scheme in clutter are smaller and less certain than in Table 6.

Table 8. Position error (m) with a cutoff of $c = 500$ m over all runs, including the lost tracks (200 runs)

Strategy	$\rho = 10^{-5}$	$\rho = 3 \times 10^{-5}$	$\rho = 10^{-4}$
S1	187.3	209.4	294.2
S2	85.0	117.1	156.0
S3	81.5	110.0	140.6
S3'	62.8	113.0	170.6
S4	104.9	160.8	282.3
S5	58.7	94.5	155.2
S6	—	80.5	—
S7	69.0	95.4	156.3
S8	43.1	83.0	123.6
S9	41.0	97.7	104.6
S10	43.3	77.5	118.3
S11	35.1	51.4	69.2

The second transmitter (S3 $\rightarrow$ S8) reduces the cutoff error by 47.1% (−62.8% to −17.9%) at the lowest density, but the intervals include zero at the middle and highest densities (−24.6%, −52.2% to 9.1%; −12.1%, −32.4% to 11.2%), and also at the middle density with the 1000 runs of Table 7, where the change, −12.1% (−27.3% to 4.8%), happens to equal the 200-run value at the highest density. With 1000 runs, the clutter factor reduces the cutoff error by 20.3% (−34.0% to −4.2%) with two transmitters and by 24.0% (−33.9% to −13.0%) with one. Some orderings change; at the lowest density, for

example, S3' has a lower cutoff error than S3 (change from S3' to S3 29.7%, -19.4% to 98.1% , not significant).

Fig. 6 shows how often S8 selects each type of estimate. About half of the scans use $\mathbf{y}_1$ or $\mathbf{y}_2$ with transmitter 2, and $\mathbf{y}_4$ is selected in about a third of the scans without clutter and a quarter in clutter. The second transmitter therefore helps in two ways: through new single-transmitter estimates on channels with higher SNR, and through $\mathbf{y}_4$, which needs no angle. However, without $\mathbf{y}_4$ (S10) the selection finds other estimates of almost the same quality, so its frequent selection brings only a small gain.

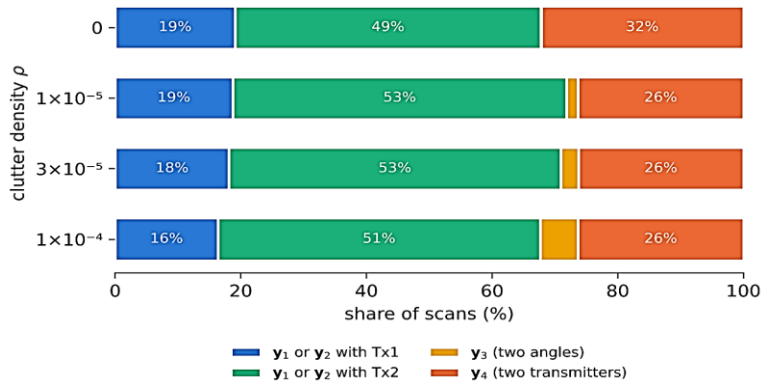


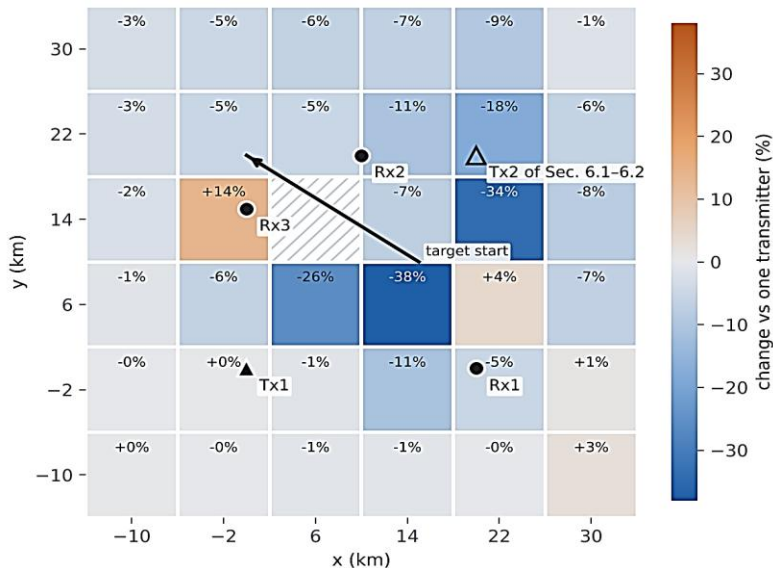
Fig. 6. Share of the scans in which each type of estimate is selected by the proposed strategy.

6.3 Effect of the Node Positions

The effect of the second transmitter location was evaluated on a 6×6 grid with a spacing of 8 km. One of the 36 cells lies within 2 km of the target path and was not simulated (hatched in Fig. 7), which leaves 35 positions. At $\rho = 10^{-5}$, 100 paired runs were used at each position. The intervals are paired bootstrap intervals for the transmitter map and bootstrap intervals of the pooled RMSE for the receiver map. The benefit of the second transmitter depends strongly on its location. The largest improvements, ranging from -18% to -38% , occur near the beginning of the target path, at $[6,6]^T$, $[14,6]^T$, $[22,14]^T$, and $[22,22]^T$ km. Of the 35 tested positions, 16 show a significant improvement, but only 5 show a significant improvement of 10% or more. In the western half of the map,

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changes are mostly between 0 and -5% . No position shows a large, statistically significant deterioration. The largest increase, $+14\%$ next to receiver 3, is not significant (-5.5% to $+36\%$). The selection can ignore a poorly placed transmitter, but only a well-placed one helps.

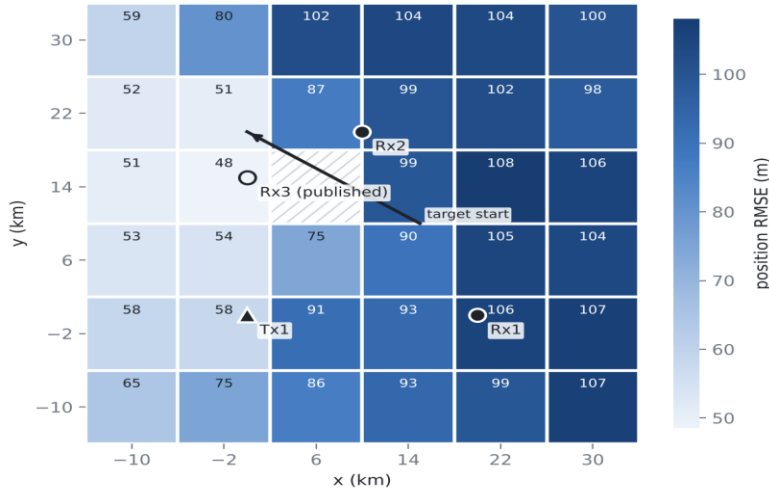


Hatched cells: not simulated (within 2 km of the target path or of another node).

Fig. 7. Change in position RMSE when a second transmitter is added at each cell (negative values are improvements). The axis ticks mark the cell centres.

Fig. 8 shows the RMSE with one transmitter when the third receiver is moved over the same grid. The RMSE ranges from 48.5m (44m to 53m) for a receiver at $[-2, 14]^T$ km to 108m (101m to 116m) at $[22, 14]^T$ km. The receiver position of Khalifa and Abousetta (2025), $[0, 15]^T$ km, lies in the best region. Moving a single receiver therefore changes the error by more than a factor of two, which motivates treating the receiver positions as a design variable, as in Liang et al. (2021) and Khalifa and Abousetta (2026).

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Hatched cells: not simulated (within 2 km of the target path or of another node).

Fig. 8. Position RMSE with one transmitter for each position of the third receiver. The axis ticks mark the cell centres.

As a further check, the comparison of S3 and S8 was repeated on a second path, with initial velocity $[-150, -100]^T$ m/s as in Nguyen and Doğançay (2019), limited to 60s because the target reaches transmitter 1 at 100s. The second transmitter reduced the position RMSE by 18.3% (-22.3% to -14.6%) without clutter and by 21.4% (-31.1% to -9.8%) at $\rho = 3 \times 10^{-5}$, close to the values on the main path.

6.4 Limitations

This study has several limitations. It uses two-dimensional simulation with a single target; the node positions are known, the nodes are synchronized, and the waveforms of different transmitters do not interfere. Clutter is uniform and independent from scan to scan, and the detection probability is constant. The selection looks one scan ahead, the clutter factor of an estimate is the product of its channel factors, and candidates that share a detection are treated as independent. The thresholds and gates were chosen by trial, although Section 3.3 shows that moderate changes of the thresholds have little effect. Each scan uses two channels, and the Cartesian estimate keeps four of their six measurements, which costs accuracy against a nonlinear filter on the raw measurements. With one receiver configuration and two target paths, the results show the size and direction of the effects rather than general performance values.

7. Conclusions

This paper extended joint waveform and Cartesian estimate selection to several transmitters and to tracking in clutter, with a new two-transmitter estimate, its UT covariance, a nonparametric PDA filter and a clutter factor in the selection cost.

The simulations show that adding a second transmitter clearly improves tracking accuracy. Without clutter, the position RMSE decreased by about 20%. In clutter, over the tracks kept, the reduction was about 22% and 30% at the lowest and highest densities (200 runs) and 21% at the middle density (1000 runs); when lost tracks are counted through the cutoff error, the reduction remains clear only at the lowest density. Most of the gain comes from the single-transmitter estimates on the channels of the second transmitter; the new estimate y_4 is often selected but adds only about 1% in position and 5% in velocity. The gains depended on using the UT covariance: with the first-order covariance, 10.5% of the tracks were lost. Accounting for clutter in the selection cost reduced the position RMSE by up to 30% and reduced track losses in the one-transmitter case. A third transmitter added little, because each scan still uses two channels. An unscented PDA filter on the raw measurements of the same two channels, chosen by the proposed selection, was 11% to 34% more accurate than the proposed scheme. Since both use the same channels, the Cartesian approach saves no communication relative to this filter; its advantages are the linear update and a selection cost in closed form, on which the filter itself relies, not the highest accuracy. Geometry also matters strongly: the benefit of the second transmitter changed with its location, and moving one receiver changed the error by more than a factor of two. Future work will examine selection costs for nonlinear filters, joint waveform and node-position selection, multiple targets, and three-dimensional tracking.

References

- Asif, A., Kandeepan, S., & Evans, R. J. (2023). Passive radar tracking in clutter using range and range-rate measurements. *Sensors*, 23(12), Article 5451.
<https://doi.org/10.3390/s23125451>

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Target Tracking with Multiple Transmitters in Clutter
<http://www.doi.org/10.62341/istj-vol39-2-ra21>

- Bar-Shalom, Y., Li, X. R., & Kirubarajan, T. (2001). Estimation with applications to tracking and navigation. John Wiley & Sons.
- Bar-Shalom, Y., Daum, F., & Huang, J. (2009). The probabilistic data association filter: Estimation in the presence of measurement origin uncertainty. *IEEE Control Systems Magazine*, 29(6), 82–100.
<https://doi.org/10.1109/MCS.2009.934469>
- Bordonaro, S., Willett, P., Bar-Shalom, Y., & Luginbuhl, T. (2019). Converted measurement sigma point Kalman filter for bistatic sonar and radar tracking. *IEEE Transactions on Aerospace and Electronic Systems*, 55(1), 147–159.
- Dhulashia, D., & Ritchie, M. A. (2024). Multistatic radar distribution geometry effects on parameter estimation accuracy. *IET Radar, Sonar & Navigation*, 18(1), 7–22.
<https://doi.org/10.1049/rsn2.12529>
- Efron, B., & Tibshirani, R. J. (1993). An introduction to the bootstrap. Chapman & Hall.
- Farina, A. (1986). Tracking function in bistatic and multistatic radar systems. *IEE Proceedings F (Communications, Radar and Signal Processing)*, 133(7), 630–637.
<https://doi.org/10.1049/ip-f-1.1986.0100>
- Feng, X., Sun, P., Liang, M., Wang, X., Zhao, Z., & Zhou, Z. (2024). Radar waveform selection for maneuvering target tracking in clutter with PDA-RBPF and Max-Q-based criterion. *Remote Sensing*, 16(11), Article 1925.
<https://doi.org/10.3390/rs16111925>
- Godrich, H., Haimovich, A. M., & Blum, R. S. (2010). Target localization accuracy gain in MIMO radar-based systems. *IEEE Transactions on Information Theory*, 56(6), 2783–2803.
<https://doi.org/10.1109/TIT.2010.2046246>
- Haimovich, A. M., Blum, R. S., & Cimini, L. J. (2008). MIMO radar with widely separated antennas. *IEEE Signal Processing Magazine*, 25(1), 116–129.
<https://doi.org/10.1109/MSP.2008.4408448>
- Huang, J., Xie, J., Zhang, H., Li, Z., & Qi, C. (2024). A novel tracking algorithm based on waveform selection for



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Target Tracking with Multiple Transmitters in Clutter
<http://www.doi.org/10.62341/istj-vol39-2-ra21>

- maneuvering targets in clutter. *Circuits, Systems, and Signal Processing*, 43(5), 3160–3179.
<https://doi.org/10.1007/s00034-024-02601-9>
- Julier, S. J., & Uhlmann, J. K. (2004). Unscented filtering and nonlinear estimation. *Proceedings of the IEEE*, 92(3), 401–422. <https://doi.org/10.1109/JPROC.2003.823141>
- Kershaw, D. J., & Evans, R. J. (1994). Optimal waveform selection for tracking systems. *IEEE Transactions on Information Theory*, 40(5), 1536–1550.
<https://doi.org/10.1109/18.333866>
- Kershaw, D. J., & Evans, R. J. (1996). A contribution to performance prediction for probabilistic data association tracking filters. *IEEE Transactions on Aerospace and Electronic Systems*, 32(3), 1143–1148.
<https://doi.org/10.1109/7.532274>
- Kershaw, D. J., & Evans, R. J. (1997). Waveform selective probabilistic data association. *IEEE Transactions on Aerospace and Electronic Systems*, 33(4), 1180–1188.
<https://doi.org/10.1109/7.625110>
- Khalifa, R. A. M., & Abousetta, M. (2025). Enhancing target tracking accuracy in multistatic radar systems using an adaptive algorithm for waveform and Cartesian estimate selection. In *Proceedings of the 10th International Conference on Control Engineering & Information Technology (CEIT-2025)* (Proceedings Book Series, Vol. 23, pp. 270–277).
- Khalifa, R. A. M., & Abousetta, M. (2026). Joint adaptive waveform selection and trajectory optimization for enhanced target tracking in mobile multistatic radar systems. In *2026 IEEE 5th International Maghreb Meeting of the Conference on Sciences and Techniques of Automatic Control and Computer Engineering (MI-STA)* (pp. 830–833). IEEE.
<https://doi.org/10.1109/MI-STA68962.2026.11511196>
- Lerro, D., & Bar-Shalom, Y. (1993). Tracking with debiased consistent converted measurements versus EKF. *IEEE Transactions on Aerospace and Electronic Systems*, 29(3), 1015–1022. <https://doi.org/10.1109/7.220948>

Joint Waveform and Cartesian Estimate Selection for Multistatic
Target Tracking with Multiple Transmitters in Clutter
<http://www.doi.org/10.62341/istj-vol39-2-ra21>

- Liang, J., Huan, M., Deng, X., Bao, T., & Wang, G. (2021). Optimal transmitter and receiver placement for localizing 2D interested-region target with constrained sensor regions. *Signal Processing*, 183, Article 108032.
<https://doi.org/10.1016/j.sigpro.2021.108032>
- Nguyen, N. H., Doğançay, K., & Davis, L. M. (2015a). Adaptive waveform and Cartesian estimate selection for multistatic target tracking. *Signal Processing*, 111, 13–25.
<https://doi.org/10.1016/j.sigpro.2014.11.022>
- Nguyen, N. H., Doğançay, K., & Davis, L. M. (2015b). Adaptive waveform selection for multistatic target tracking. *IEEE Transactions on Aerospace and Electronic Systems*, 51(1), 688–701. <https://doi.org/10.1109/TAES.2014.130723>
- Nguyen, N. H., & Doğançay, K. (2019). Signal processing for multistatic radar systems: Adaptive waveform selection, optimal geometries and pseudolinear tracking algorithms. Elsevier.
- Schuhmacher, D., Vo, B.-T., & Vo, B.-N. (2008). A consistent metric for performance evaluation of multi-object filters. *IEEE Transactions on Signal Processing*, 56(8), 3447–3457.
<https://doi.org/10.1109/TSP.2008.920469>
- Shi, C., Wang, Y., Salous, S., Zhou, J., & Yan, J. (2022). Joint transmit resource management and waveform selection strategy for target tracking in distributed phased array radar network. *IEEE Transactions on Aerospace and Electronic Systems*, 58(4), 2762–2778.
<https://doi.org/10.1109/TAES.2021.3138869>
- Thornton, C. E., Buehrer, R. M., Dhillon, H. S., & Martone, A. F. (2022). Universal learning waveform selection strategies for adaptive target tracking. *IEEE Transactions on Aerospace and Electronic Systems*, 58(6), 5798–5814.
<https://doi.org/10.1109/TAES.2022.3181554>
- Wang, S., & Bao, Q. (2021). Decorrelated unbiased converted measurement for bistatic radar tracking. *Journal of Applied Remote Sensing*, 15(1), Article 016507.
<https://doi.org/10.1117/1.JRS.15.016507>

Joint Waveform and Cartesian Estimate Selection for Multistatic
Target Tracking with Multiple Transmitters in Clutter
<http://www.doi.org/10.62341/istj-vol39-2-ra21>

- Wang, Y., Shi, C., Wang, F., & Zhou, J. (2021). Collaborative transmit resource scheduling and waveform selection for target tracking in multistatic radar system. *IET Radar, Sonar & Navigation*, 15(3), 209–225.
<https://doi.org/10.1049/rsn2.12026>
- Willis, N. J., & Griffiths, H. D. (Eds.). (2007). *Advances in bistatic radar*. SciTech Publishing.
- Zhu, Y., Zhao, L., Zhang, Y., & Wu, X. (2021). Receiver selection for multi-target tracking in multi-static Doppler radar systems. *EURASIP Journal on Advances in Signal Processing*, 2021, Article 118. <https://doi.org/10.1186/s13634-021-00826-3>.